Summary: Cross-Disciplinary Sessions

P.H. Diamond

UCSD and SWIP

AAPPS-DPP (菊池祭)
Kanazawa, 2018

- N.B.: 2018 is 40th Anniversary of Hasegawa-Mima Classic paper → Inspired many cross-disciplinary bridges...

- Special Thanks:

  Prof. Yoshi-Yuki Hayashi, co-organizer

Outline

- Inventory of papers
- Highlights:
  - Flows
  - Magnetic Self-Organization
  - Turbulence dynamics
  - Assorted Topics
- What did we learn? – Discussions
  - Plasma-GFD Interactions → Future?
  - Magnetic Self-Organization and Dynamo – Status?
  - New Directions in Nonlinear Dynamics in Plasma?

Inventory

Inventory

- 4 Plenary + 30 Submitted Papers
- A) Flow Self-Organization and Transport (GFD and Plasmas)
  - Structure Formation: Yoden, Niino, Takehiro, Arakawa, Yim, Kosuga, Yamada
  - Momentum Transport: M. Yamada, Hayashi, Ko, Cao, Aiki
  - Turbulence and Transport: Lathrop, Guo, Kanik, Noh, Kawamori, Iwayama
- Discussion: Directions for future CD interaction of plasma and GFD communities?!

Inventory, cont'd

- B) Magnetic Self-Organization: Dynamo, Relaxation, Reconnection (Structure Formation ←→ Magnetic Fields)
  - Relaxation: Cappello, Chen, Zhang, Singh
  - Dynamo: Tobias, Deguchi, Hori
  - Reconnection and Islands: Xiao, Sydora, Jiang
- Discussion: Magnetic Self-Organization and Dynamo: Current Status and Role of Boundary

Inventory, cont'd

- Assorted Topics: Mostly Laser-Plasma

  Huang, Wang, Shiroto, Miyoshi, Liu

- Discussion: New Directions in Nonlinear Plasma Dynamics!?

Highlights

- Flow

The cross-disciplinary session…

- Analogy between GFD & tokamak plasmas
  - Waves…
  - Vortices…
  - Staircasing
  - Interactions with mean flows
  - Quasi two-dimensionality
  - Mixing of Lagrangian conserved quantities…
  - Transport barriers

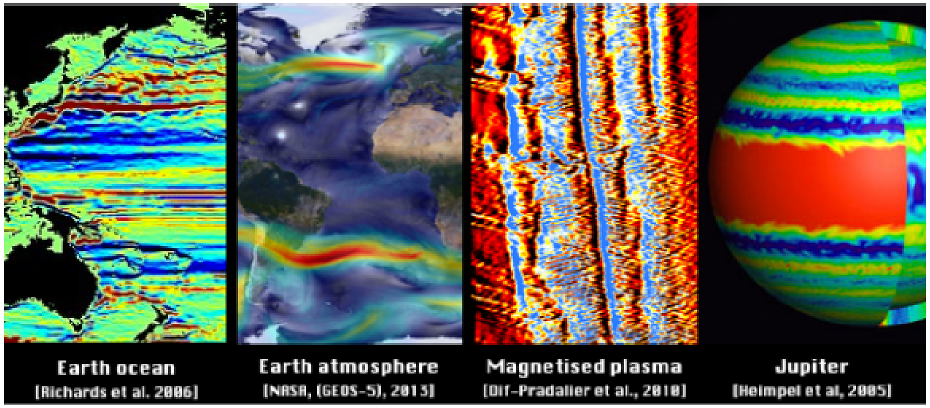


Figure: G. Dif-Pradalier

$$\frac{D\,q}{D\,t} = 0$$

$$q = \nabla^2\psi + \beta\,y - F\,\psi$$

CHM equation

$$\frac{\partial}{\partial t}(\nabla^2\psi - F\psi) + \beta\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x} = 0\,.$$

- *Can we break this analogy?*

Hayashi: Turbulence, waves, momentum transfer in GFD

- Model reduction in GFD from Richardson, 1922
  - Numerical weather prediction with primitive computers → forced reduction, like for MHD - "mission orientation"
- Theoretical structure

  a) Stability theorems: Rayleigh, Lin, Fjorthoft

  b) Equations/models: Charney → QG → H-M

  - Waves (Rossby), Coherent structure → vortex
  - Potential vorticity $dq/dt = 0$
    - Conserved phase space density (ala' Vlasov equation)
    - Dual cascade, inverse energy (Kraichnan)

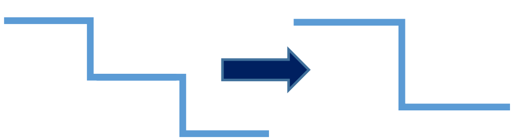
Hayashi: Turbulence, waves, momentum transfer in GFD, cont'd

c)

- Weakly nonlinear wave theory
- Pseudo-momentum, wave-flow interaction → jets! → $u^{(2)} = -\overline{\varepsilon'^2}/2Q_y$

d) <u>Problematic</u>

- Turbulence
- Convection – multi-scale, break PV cons
- Scale selection
  - Staircases
  - "Wave-flow jigsaw puzzles"
  - Emergent: Rhines → Major current focus
- Jet mergers



but on exp.
time scale

Tornadoes: Their Structure, Genesis Mechanism and Environment

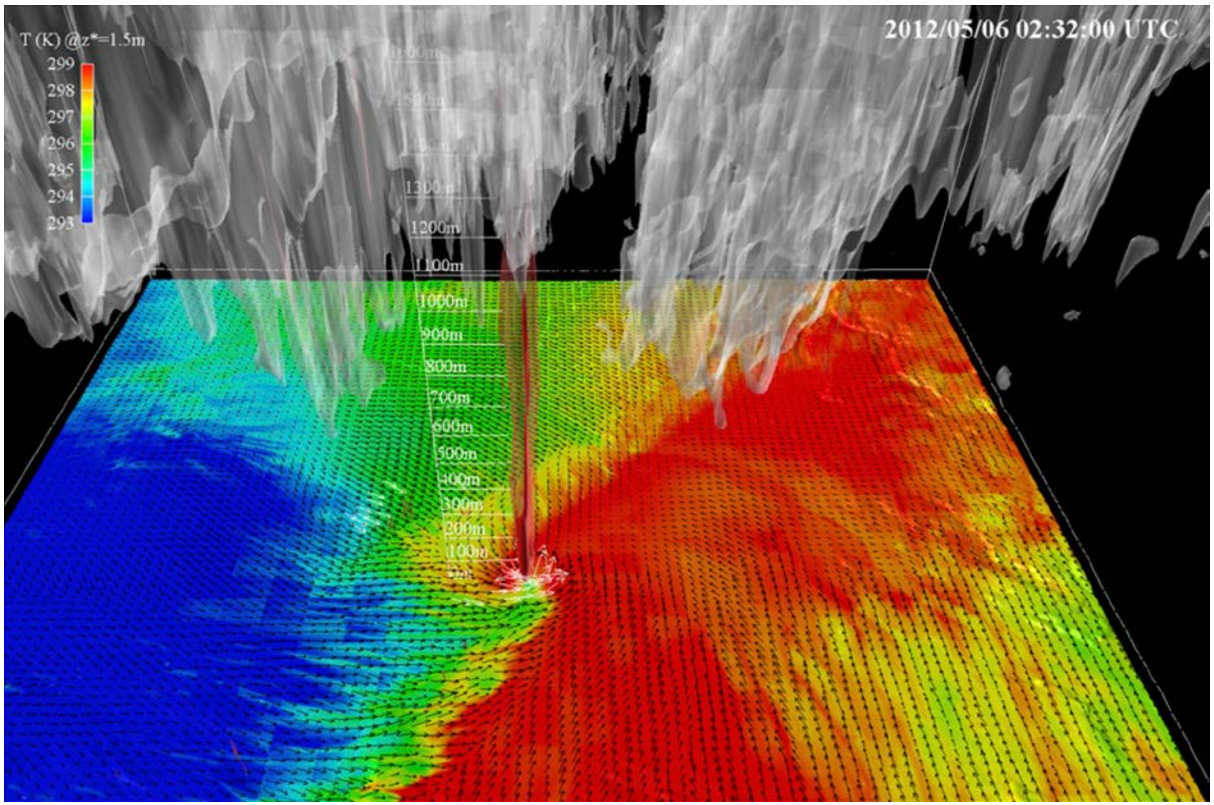
Hiroshi Niino (Atmosphere and Ocean Research Institute, The University of Tokyo)

* Our current understanding of tornadoes are reviewed, and future subjects are discussed.

* Tsukuba, Japan F3 tornado on 6 May 2012 was successfully reproduced by a quadruply-nested 32 member ensemble simulation. An ensemble sensitivity analysis shows that the origins of rotation of tornadoes vary from member to member, and the strength of tornado is highly correlated with that of low-level mesocyclone.

* The multiple-vortex structure of the same tornado was successfully reproduced by a simulation with 10m horizontal mesh [Mashiko and Niino, SOLA, 2017].

* It is important to recognize a hierarchy of atmospheric disturbances that causes tornadoes, and Storm-Relative Environmental Helicity and Entraining Convective Available Potential Energy of the environment seem to be important parameters to assess a potential risk of tornadoes [Sueki and Niino, GRL,2016; Tochimoto and Niino, MWR, 2016, 2018 submitted].



Three-dimensional structure of the simulated tornadic vortex. Red translucent and opaque isosurfaces show vertical vorticities of 0.2 and 0.6 s^{-1}, respectively, gray isosurfaces cloud water mixing ratio of 1 g/kg, color shading temperature (K) at 1.5 m AGL, black and white arrows horizontal winds for less and larger than 30 m/s at 30 m AGL, respectively [Yokota et al., 2018, Monthly Weather Review (MWR)].

Tornado Genesis, cont'd

- Super-cell → mesocyclone → tornado

  stochastic boundary process

- Ingredients:
  - SREH → Storm Relative Environmental Helicity
  - SREH $= -\int^{h} \vec{k} \cdot \left(\vec{v} - \vec{c}\right) \times \frac{\partial \vec{v}}{\partial z}\, dz$ <u>updrafts</u>
  - CAPE → convective available potential energy

➔ FOM : (SREH)(CAPE) → basis for tornado warning

Lathrop: Vortex Dynamics and Reconnection in Quantum Fluids

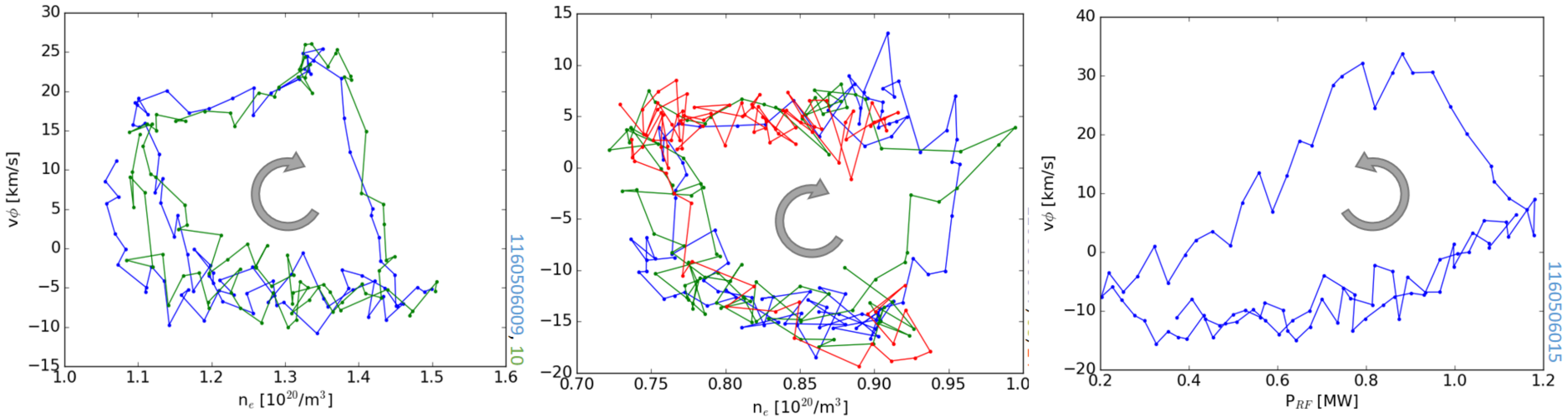
- Discussed: ideal MHD, ideal fluids, quantum fluids
- Quantum Fluid:
  - Repulsive interaction NLS
  - Waves ala' Kelvin $\omega \sim k^2$
- Singular Dynamics
  - Vortex line reconnection (via acoustics)
  - $V(t) \sim 1/(t_R - t)^{1/2}$
  - Pdf V ~ $1/V^{3/2}$ → tail, akin particle acceleration

Filament helical dynamics evident

Norman Cao: Reversals, Hysteresis and Turbulence Populations

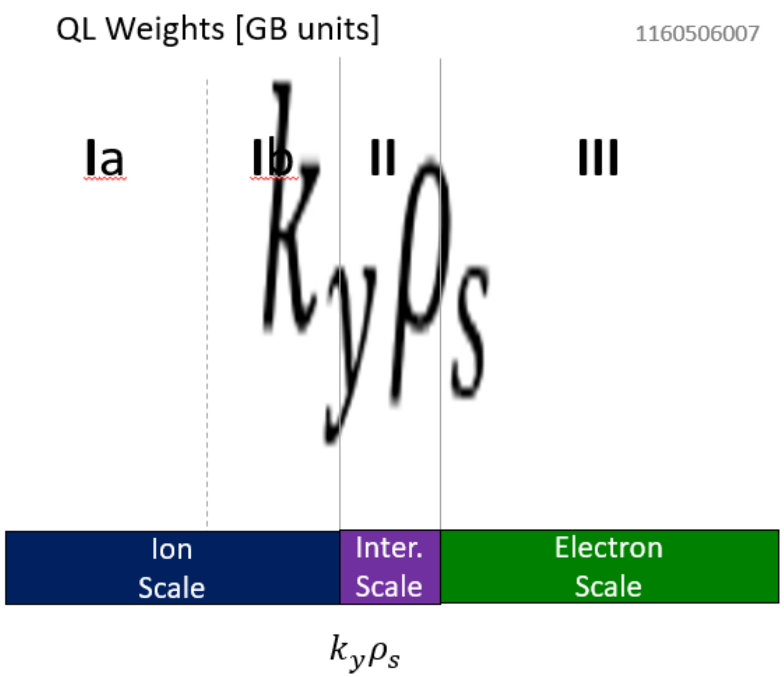
Hysteresis Observed Robustly in Multiple Plasma Conditions

- Under different plasma conditions, transition appears to occur when the normalized collisionality crosses $\nu^* \equiv \frac{\nu_e \epsilon}{\omega_{be}} \approx 0.4$ [Rice NF 2013]
  - Suggests the link with trapped electron modes



Norman Cao: Reversals, Hysteresis and Turbulence Populations

Subdominant Mode Transition Found to be Consistent with Observed Transport



- In order to satisfy particle flux constraint, two solutions exist:

	SOC-like	LOC-like
Active Mode Families	ITG (Ib) ETG (III)	ITG (Ia, Ib) TEM-like (II) ETG (III)
Particle Flux Balance	Balance within Ib	Ia balances II

- Leading to qualitatively different transport dependencies

Electron Heat Transport	ETG dominates	TEM and ETG
Torque Balance	ITG dominates	TEM and ITG

Norman Cao: Reversals, Hysteresis and Turbulence Populations

Conclusions and Future Work

- Experiments show **changes in toroidal rotation** and turbulent residual stress despite **nearly identical density and temperature profiles**
  - A change in dominant linear instability alone is not sufficient to explain the LOC/SOC transition
- Quasilinear modelling shows that a **subdominant ITG/TEM transition** is consistent with the observed transport
  - Reminiscent of a "population collapse" or quenching of turbulent TEM-like mode intensity
- *Future work*: Compare predictions against global nonlinear simulation, and identify if changes are consistent with fluctuations measured in experiment

Kosuga: Pattern Competition

- Zonal Flow vs Streamer Competition

- Perp-Parallel Synergy

    → ‘General Circulation’

Kosuga: Pattern Competition

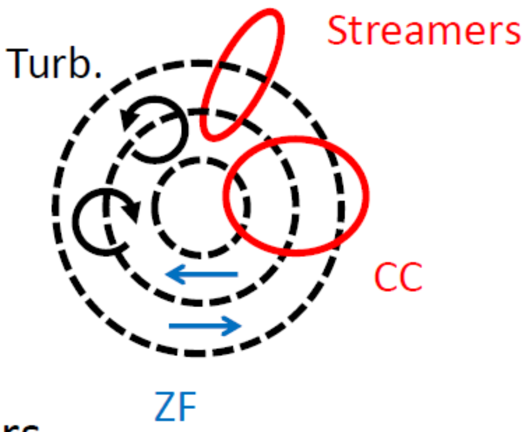
Highlights of the talk

- Difference in the driving of zonal flows/Streamers?

  Kosuga, et al., Phys. Plasmas **25** 100701 (2018)

  Conventional: shearing feedback

  -> Relevance of density mod. for streamers

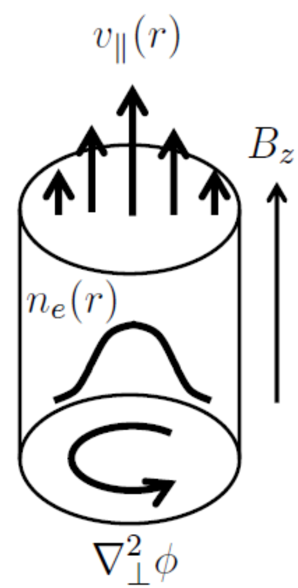


- Role of 3rd direction: parallel flows?

  Kosuga, Phys. Plasmas **24** 122305 (2017)

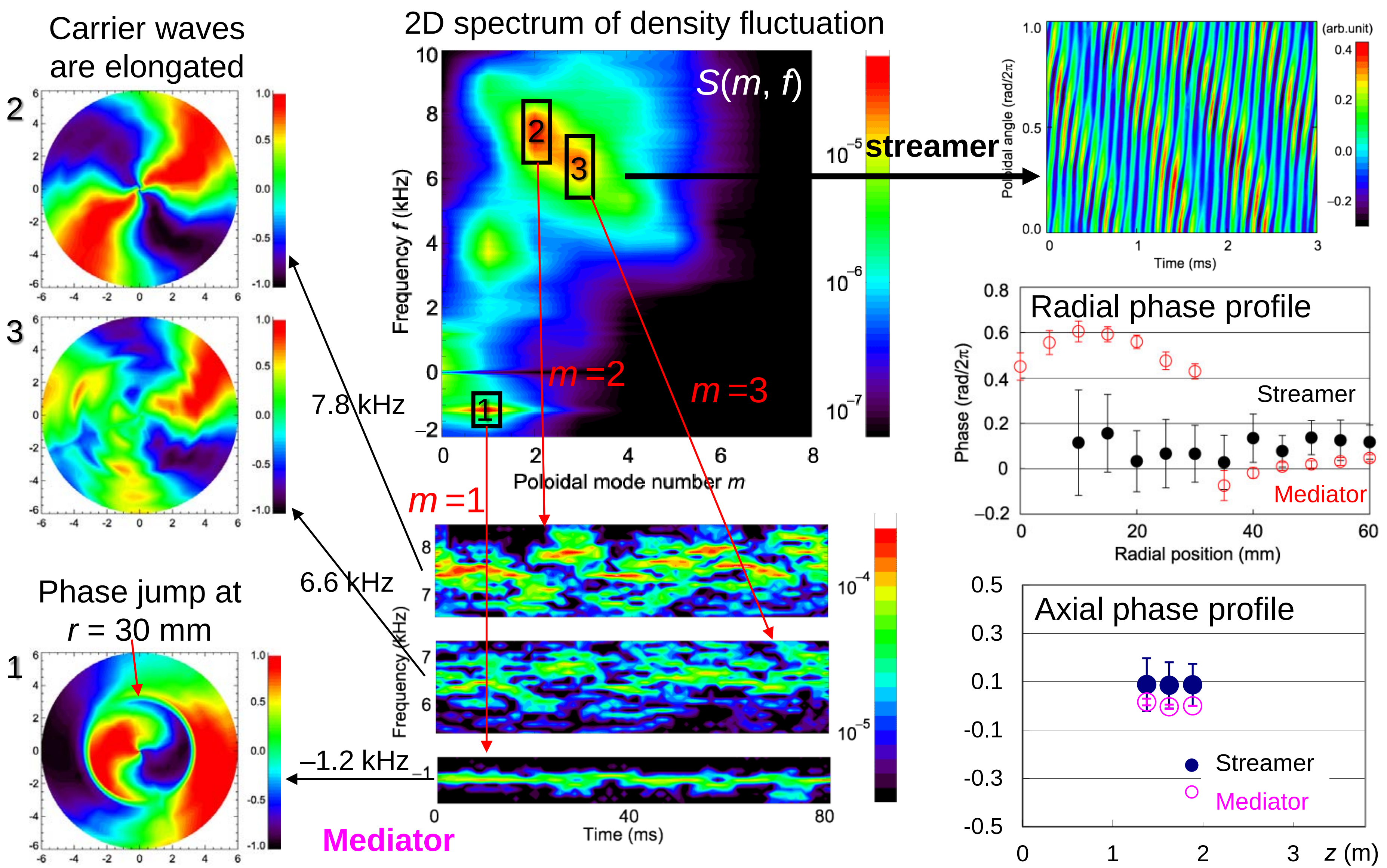
  Recent developments on parallel flows

  Interplay between perp. and parallel flows



Three dimensional structure of streamer in drift wave fluctuations (T. Yamada)

3D structures of mediator, carrier waves, and streamer have been clarified



Tobias: Weak Magnetic Field Effects on GFD

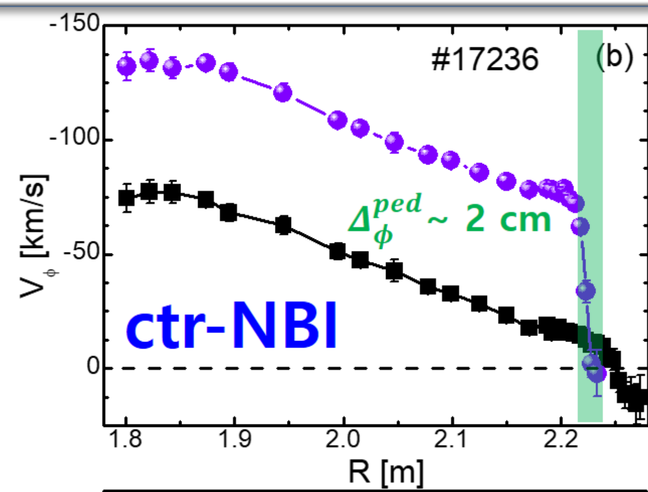
- Application: Solar Tachocline, Proto-Planets, Rapidly Rotating Stars...

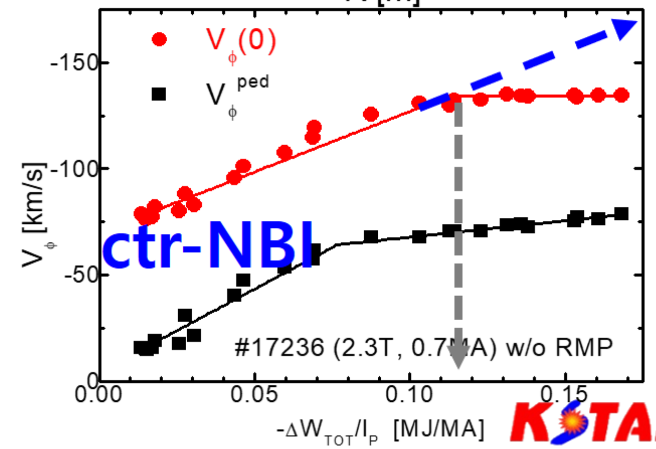
Conclusions/Future Work

- The analogy between stratified, rotating geophysical flows and plasma flows is worth exploring
- Even a small amount of large scale magnetic field can drastically change the physics of the strictly geophysical problem
  - Breaks PV conservation
  - Switches off jets
  - Can disrupt vortices
  - Subtle Lagrangian effects (cf Vainshtein & Cattaneo 1991)
- Does this carry across to more elaborate geophysical models
  - Shallow water MHD (Cho & Staehling)
  - MHD thin shell primitive equations (Miesch 2007, McCabe & Tobias in preparation)

Rotation and momentum transport in magnetic confined plasmas by Won-Ha Ko (NFRI)

- **The rotation development is observed during sawtooth and toroidal rotation has quite unique pedestal in KSTAR which is advantage for rotation and momentum transport physics. Resonant δB reduced pedestal widths of rotation (Δ_{ϕ}^{ped}~ 5cm → 3 cm) in co-NBI and ctr-NBI decreased rotation pedestal (Δ_{ϕ}^{ped}~ 5cm → 2 cm) in w/o resonant δB while those of Ti (Δ_{i}^{ped}~ 2cm) are constant in any case**
- **A clear disparity of the core toroidal rotation in between co-NBI and counter-NBI heated H-mode plasmas gave us a clue of intrinsic rotation. The first prediction and estimation of core intrinsic flows in spite of the strong external momentum input is reported in KSTAR.**
- **Whole toroidal rotation close to zero but edge has small rotation which may be from intrinsic rotation despite the RMPs strongly attack from edge region.**
- **Observation of core and edge rotation is important role in understanding physical mechanism and its generation of intrinsic torque.**





K. Hori: B-field Effects on Waves in GFD

Summary

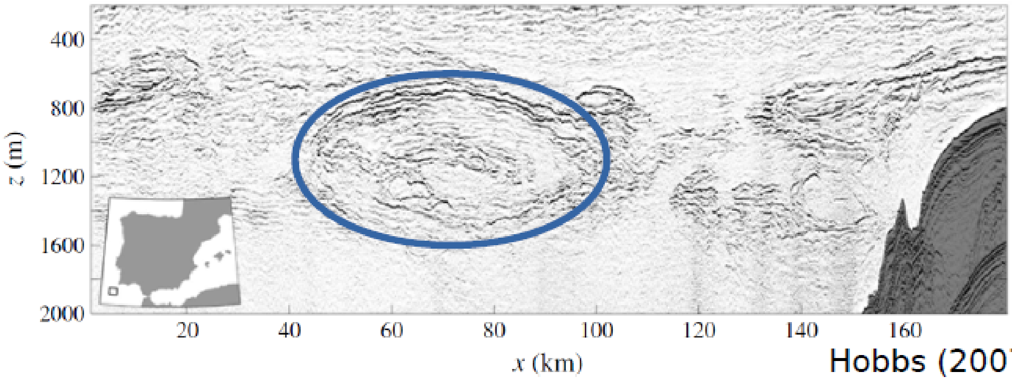
- The excitation of rotating MHD waves is supported by geo-/Jovian dynamo simulations:
  - axisymmetric, torsional Alfven waves
    - propagating in cylindrical radius with timescales of $2\pi\ \omega_M^{-1}$
      - 4-6 years in Earth's core;   possibly $\gtrsim$ 10 years in Jupiter's metallic H-He region
    - reflecting at an interface between the metallic-molecular regions
  - nonaxisymmetric, slow magnetic Rossby waves
    - crests/troughs travelling retrogradely with timescales of 2π $[\ \omega_M^2/\omega_R\ ]^{-1}$ with respect to mean zonal flows
      - on timescales of $O(10^{1\text{-}2}$ yrs) in Earth's core
    - nonlinear Lorentz terms play a role in steepening waveforms
- Detecting these wave properties could enable us to infer physical quantities of the planetary dynamos

Global stability of pancake vortices in stratified-rotating fluids

CD118, Eunok Yim

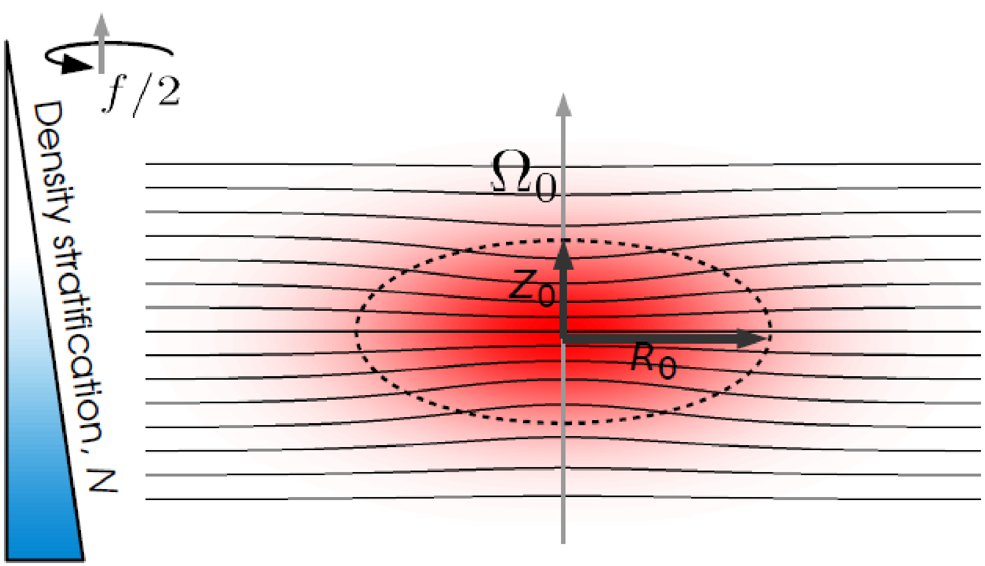
- **Motivation**
  - Mediterranean eddy

<Seismic reflectivity of Gulf of Cadiz>

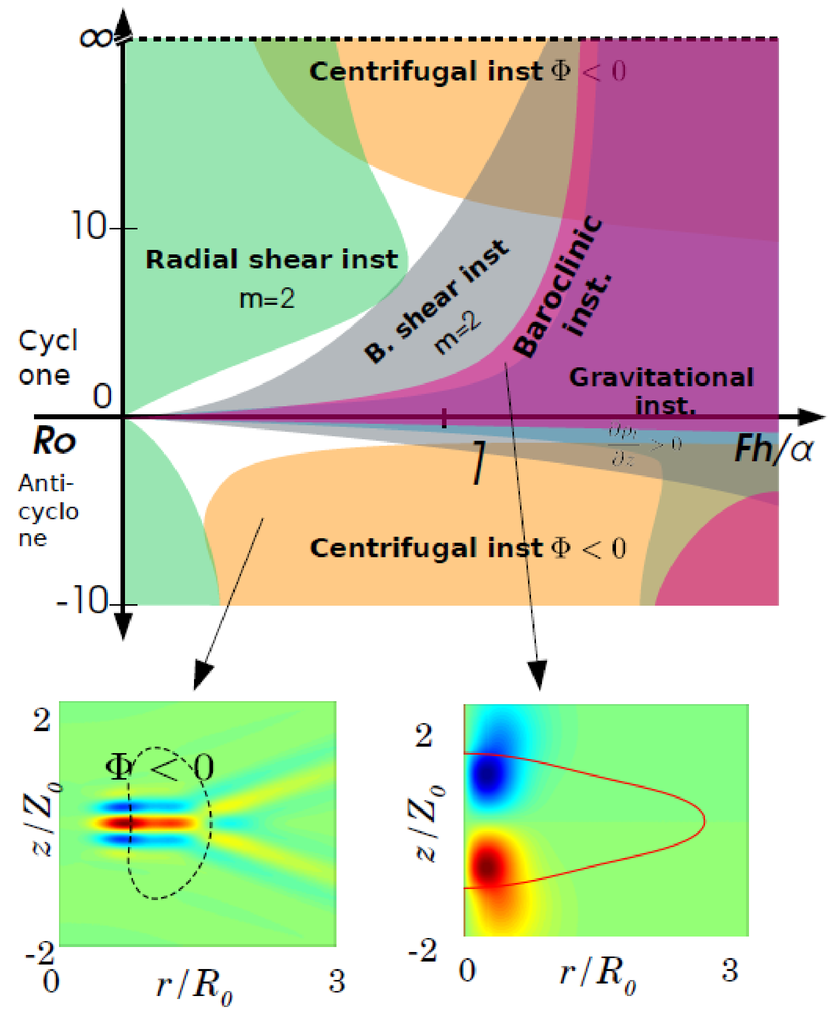


Hobbs (2007)

- **3D pancake vortex**

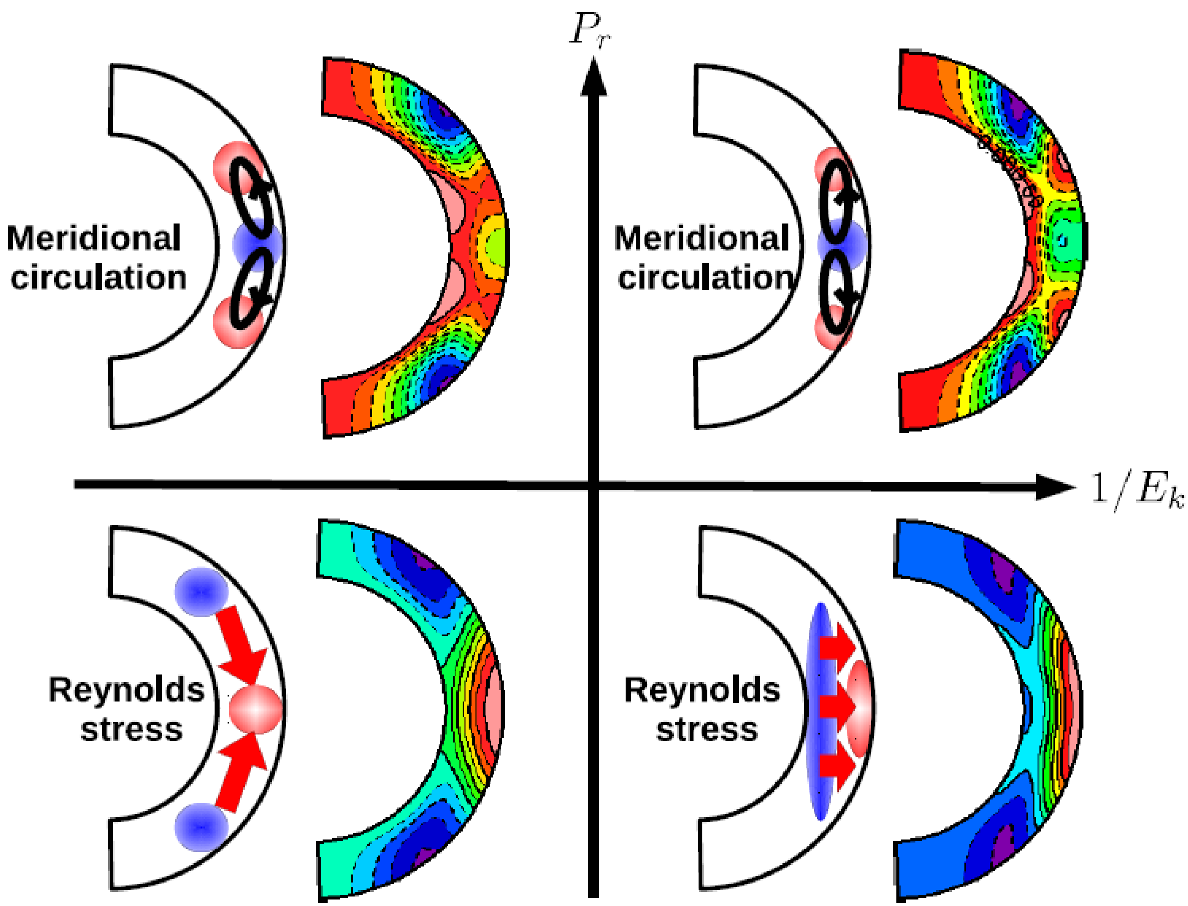


- **Stability map**



Thermal convection and induced mean zonal flows in rotating spherical shells (S. Takehiro)

Summary of weak nonlinear theory



- Large P_r : meridional circulation
  $\Downarrow$
  equatorial sub-rotation
- Small P_r : Reynolds stress
  $\Downarrow$
  equatorial super-rotation

CD-I16: 2018.11.14 Wed 15:15-15:40

Towards a seamlessly diagnosable expression for the energy flux associated with both equatorial and mid-latitude waves

Nori Aiki (ISEE, Nagoya Univ., Japan)

A theoretical gap in traditional formulation
… challenge for tropical-extratropical interaction analysis

without relying on Fourier analysisnor Ray theory

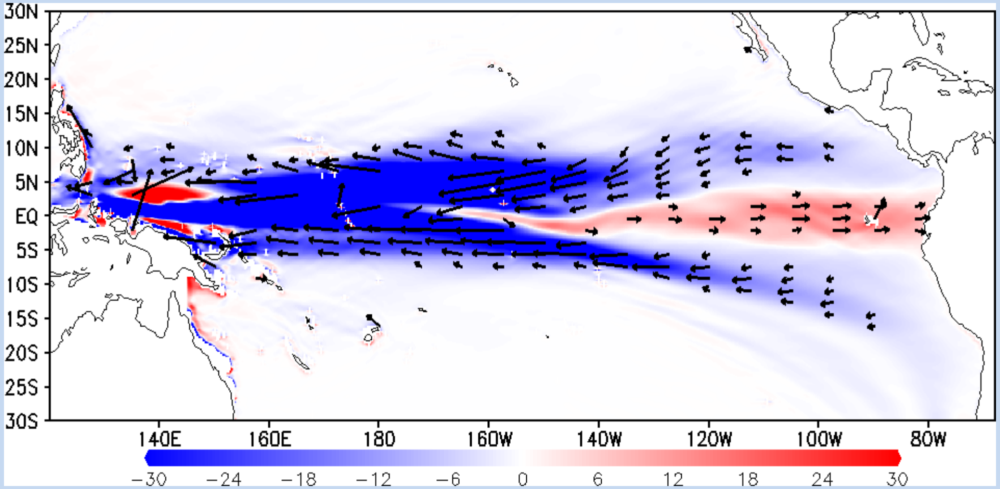
How to estimate group-velocity-based energy flux from model output for waves at all latitudes?

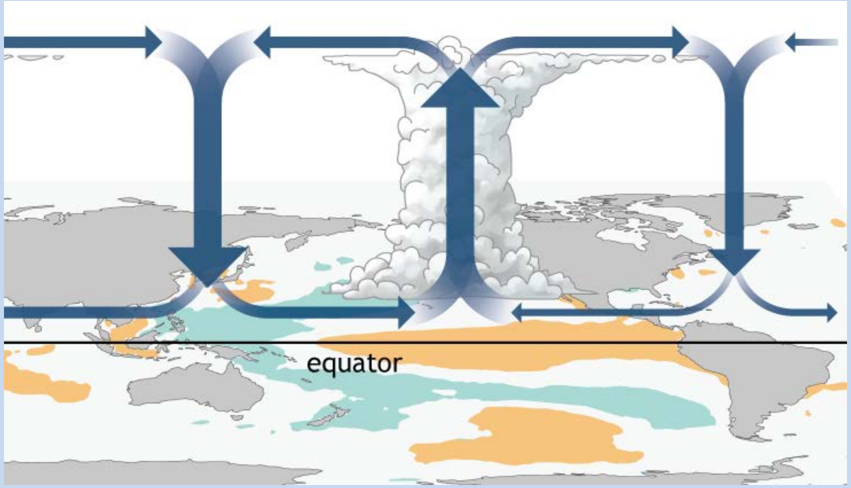
New inversion of Ertel's potential vorticity
… auto-focus for all gravity and planetary waves at all latitudes

$$\nabla^2\varphi - (f/c)^2\varphi - (3/c^2)\varphi_{tt} = q'$$

$$\partial_t \overline{E} + \nabla \cdot \langle\langle \underbrace{\overline{u'p'} + (\overline{p'\varphi}/2 + \overline{u'_{tt}\varphi}/\beta)_y}_{=c_g\overline{E}}, \overline{v'p'} - (\overline{p'\varphi}/2 + \overline{u'_{tt}\varphi}/\beta)_x \rangle\rangle = 0$$

Aiki et al. (2017 Progress of Earth & Planetary Science)





A future direction of Geophysical Fluid Dynamics
The life-cycle analysis of wave energy in the global atmosphere and ocean (i.e. wave tracing based on group velocity) …
… for clarifying the mechanism of tropical-extratropical interaction associated with El Nino / La Nina

Highlights

- Magnetic

Magnetic Self-Organization

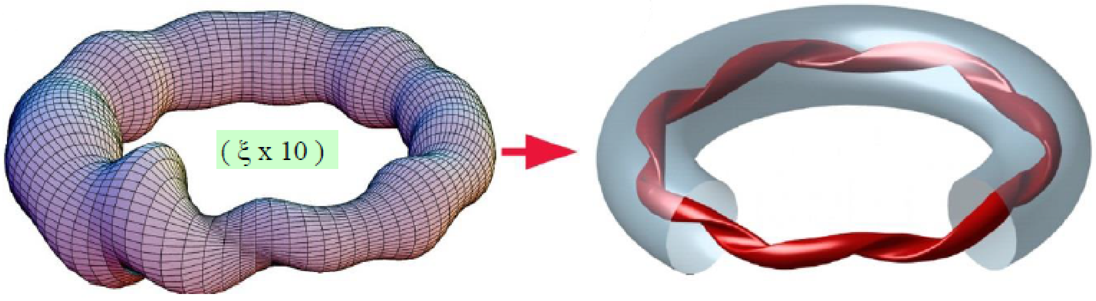
- Taylor Relaxation – rooted in dual cascade of energy, magnetic helicity – is over-arching theme for macroscopics
- Microscopic mechanism less clear
- Important ‘exceptions’, ‘unusual cases’ emerging

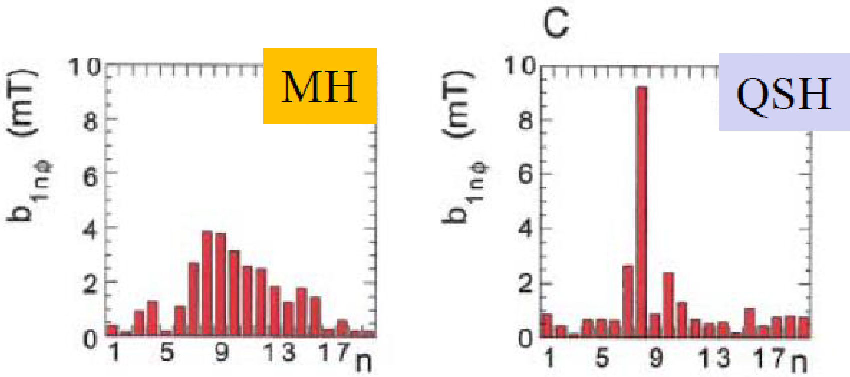
RFP self-organization

Experimental overview

RFP ↔ saturated KINKED plasma

for Ip above ~ 1 MA

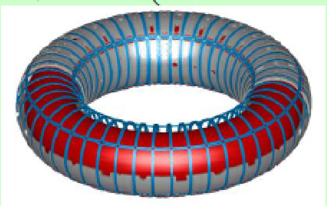




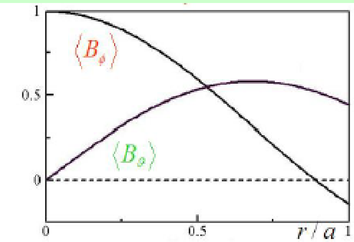
MHD spectrum: resistive kink-tearing modes

Advanced operation required in RFX-mod

CLEAN MODE CONTROL
and/or
NON CONVENTIONAL
SCENARIOS (PPCD-OPCD)



Feedback coils system
Typical operation:
Ip ~ 1.7 MA
Te up to 1.2 keV

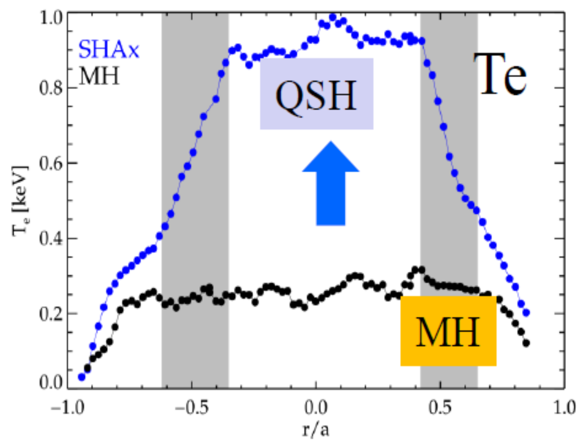


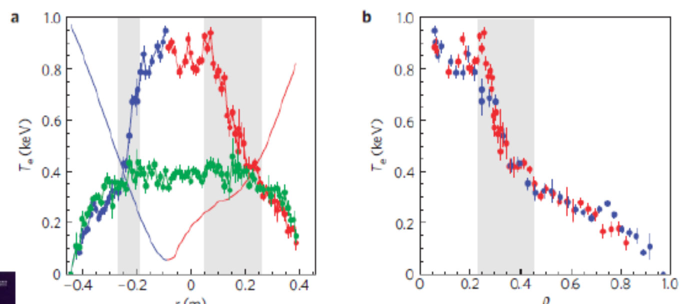
RFP helical self-organization: structures formation

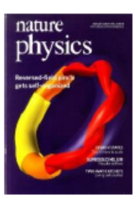
Formation of e-Transport Barriers

24598, 117ms
I_p=1.3MA
n/n_G=0.20

22201, 35ms
I_p=0.7MA
n/n_G=0.22



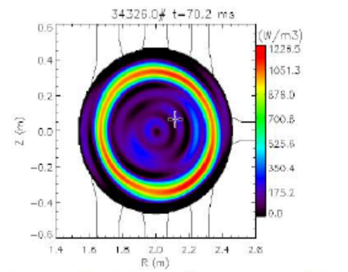




Lorenzini, Martines, Piovesan et al NatPhys 2009

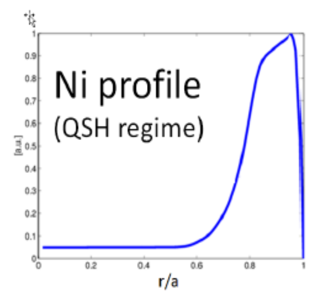
… and Impurity screening effect

41[st] EPS Conference on Plasma Physics



Tungsten LBO in QSH

Fig.2 Experimental tomographic inversion of SXR emissivity at the maximum of the W LBO emission



Simulation of Ni total density at the steady state LBO in QSH

Puiatti, Valisa, Agostini et al NF 2011
Carraro, Auriemma, Barbui et al EPS 2014
Menmuir, Carraro, Alfier et al PPCF 2010

TESPEL experiments (RFX-mod2) planned in collaboration with NIFS

Summary

Helical self-organization characterizes the Reversed Field Pinch:

- Experiments show formation of thermal and impurity particle barriers,
- 3D MHD shows magnetic chaos healing and hidden coherent structures

(Lagrangian Coherent Structures LCS),

- **New global helical regimes** stimulated by seed Magnetic Pertubations,
  - ✓ Characterized by tunable amplitude and frequency of «sawtoothing»,
  - ✓ Suggested by MHD and obtained in RFX-mod,
  - ✓ 3DMHD Non resonant modes provide more efficient chaos healing

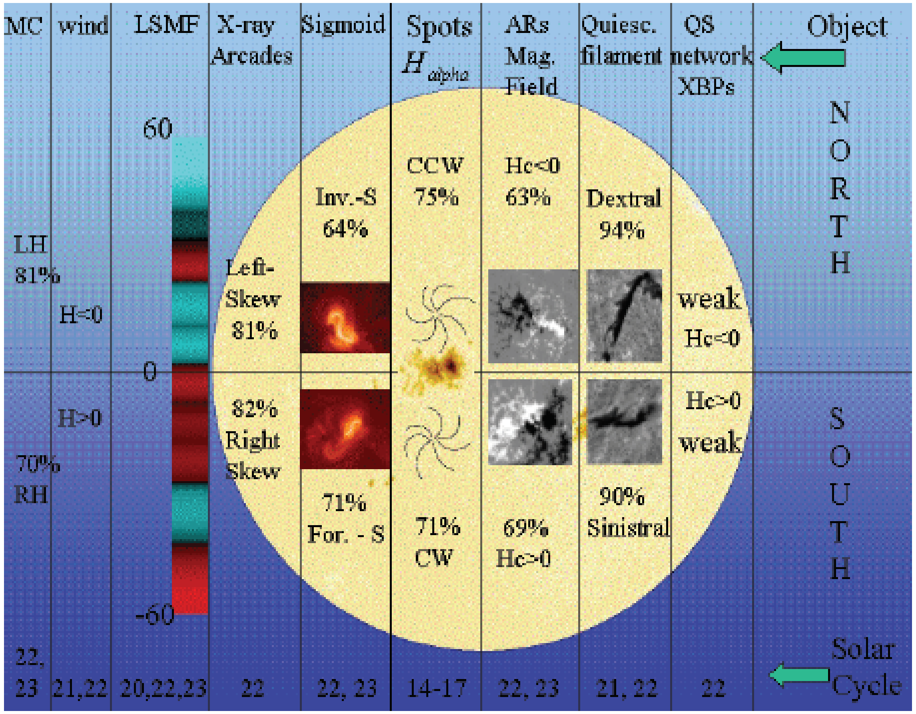
Await for further experiments in RFX-mod**2** from 2020

… we expect more effective feedback coils actions

M. Zhang: Solar Magnetic Helicity ‘Conveyor Belt’

- Helicity Limit → CME

In the corona:



1. Hemispheric helicity sign rule

(Image credit: A. Pevtsov)

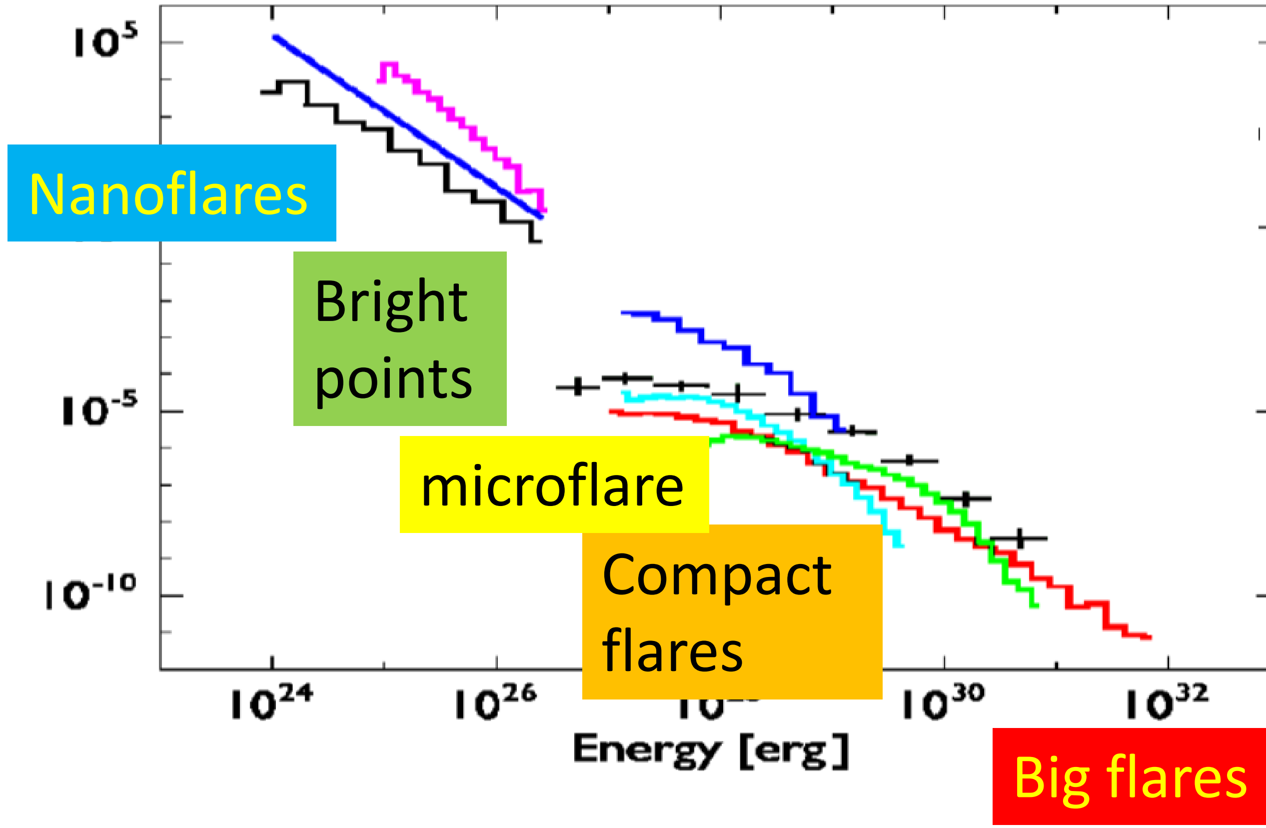
2. Berger (1984)'s conservation law

=> Magnetic helicity is accumulating in the corona !

Concluding Remarks

1. Hemispheric helicity sign rule is observed on the photosphere.
   - The rule shows solar cycle variation in sunspots.
   - Same tendency found in dynamo simulations.
2. The accumulation of magnetic helicity in the corona
   - Give rise to flux rope formation in the corona.
   - Result in CME as a natural product of coronal evolution.
   - Helicity upbound depends on boundary flux distribution.
   - Central flux rope becomes kink unstable.
3. When helicity is dumped into the interplanetary space
   - Parker-spiral-like structures will form.

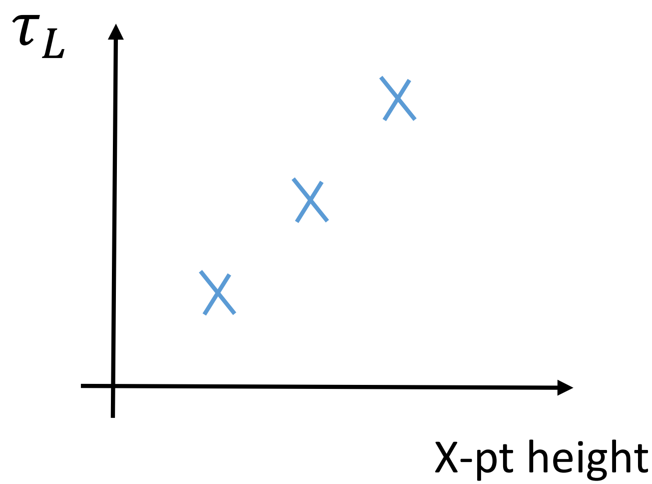
P. F. Chen



Magnetic reconnection in the solar corona is self-organized because of helicity conservation.

➡ Solar eruptions with 8 orders of dynamic range present a power-law distribution.

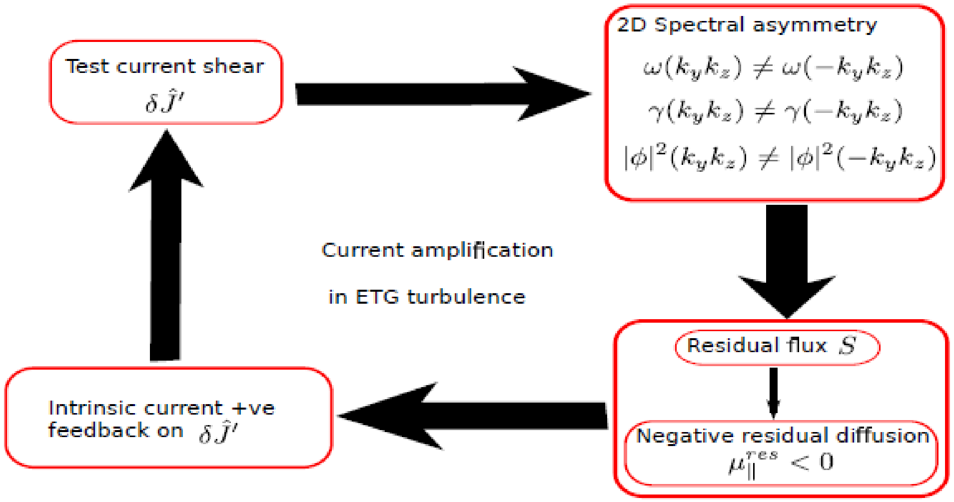
- Solar Activity as SOC
  - Magnetic Helicity – Conserved order parameter
  - $(size)^{-\alpha}$, $\tau_{wait}^{-\beta}$ distributions
- Flare
  - Plasmoid ejection



- Goal:
  - Prediction of large events – space weather
- N.B. Lathrop – exploit solar surface visualizations

Intrinsic parallel current generation from ETG turbulence in a shearless cylindrical plasma

Intrinsic current generation by residual current flux via $k_{||}$ sysmmetry breaking in electron temperature gradient turbulence is well known in sheared magnetic system like tokamaks. However this effect vanishes when magentic shear $\hat{s}$ goes to zero as $\langle k_{||} \rangle \rightarrow k_y \hat{s} \int dxx|\phi|^2$



- Test current shear asymmetrizes the growth and frequency in $k_y k_z$ space.
- $k_y k_z$ symmetry breaking converts the residual flux into diffusive flux with a -ve residual diffusivity.
- When the negative residual diffusity exceeds the ambient turbulent positive diffusivity the test axial current shear gets unstable

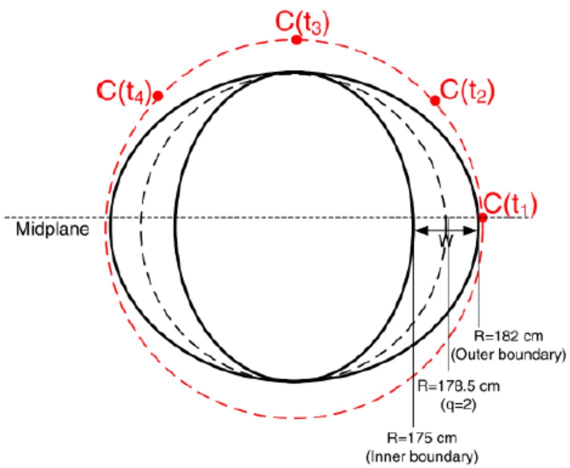
Highlights

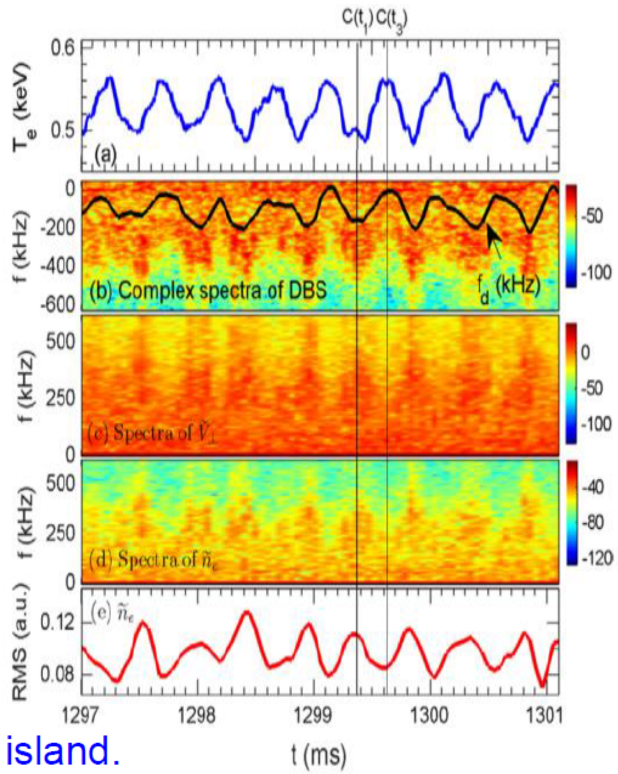
- Turbulence Dynamics

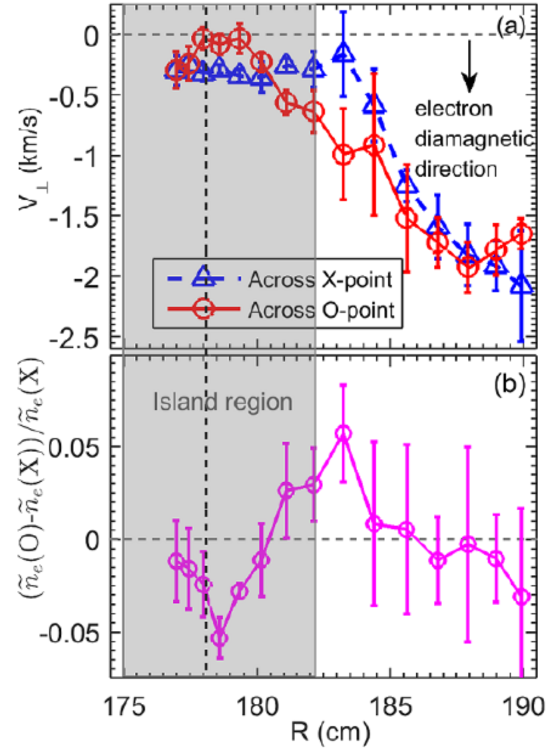
Turbulence Dynamics During Relaxation

- Reviews aspects of turbulence relevant to relaxation, self-organization
- Aspects of turbulence ubiquitous in all papers in CD session

Influence of magnetic island on $V_\perp$ and $\tilde{n}_e$







- $V_\perp$, $\tilde{V}_\perp$ and $\tilde{n}_e$ are modulated by the rotation of island.
  - ✓ Flow shear formed at boundary of island.
  - ✓ $\tilde{n}_e$ reduced inside island while enhanced at island boundary, consistent with gradient-driven turbulence.

Open issue: which one plays a more important role in regulating turbulence at the island boundary? $E \times B$ flow shear or ∇T_e?

Related:

- Z.B. Guo – Relaxation in 3 wave coupling
  - Phase modulation modifies 'Poinsot Construction' for 3 wave phase space
  - Study of trajectories for 'force free' case – 1 wave $A \equiv const.$
- Y. Noh – Turbulence in particle laden flows
  - Schmolochowski model for aerosol accumulation
  - Turbulence effects on interaction kernel

Assorted

Summary

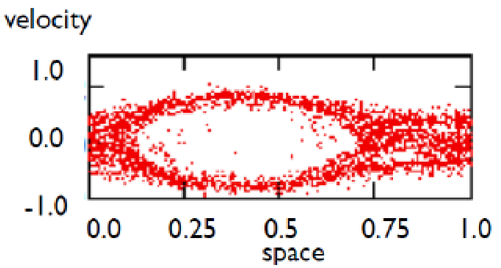
[Fujimoto, GRL, in press]

We have proposed a new theoretical model connecting the observed flow jets (i.e., BBFs) and collisionless reconnection by means of large-scale AMR-PIC.

- Formation of 3D flux ropes arising in the turbulent current layer around the x-line is a key to generate 3D outflow jets.
- The electron flow shear mode (with $\lambda \sim 20c/\omega_{pi}$) determines the cross-tail scale of the flux lopes and, therefore, the 3D reconnection jets.
- The electron dynamics plays an important role in controlling the "MHD-scale" process.

Summary of CD-O5, Takahiro Miyoshi

- The QGP, the extreme state of matter, is created in the high-energy heavy ion collision experiments
- However, the whole space-time evolution of the high-energy heavy ion collision is very complicated
- Collective dynamics of the quark matters has not been well understood yet
- In this talk, we discuss that plasma physics can contribute to the understanding of the physical processes of the high-energy heavy ion collision
  - Collisionless "glasma"
  - Relativistic MHD
  - Numerical methods

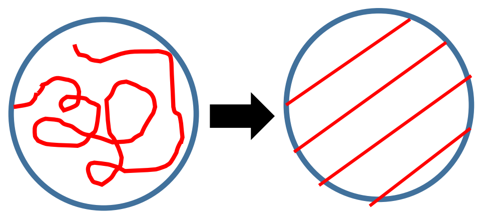


Colored Particle-In-Cell (CPIC)

What did we learn?

Discussion I: 40 years after Hasegawa-Mima Future Directions for Plasma (MFE Theory) – GFD Interactions

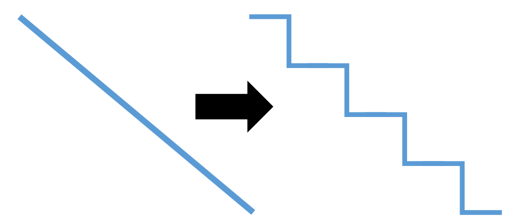
- Leaders: Diamond, Hayashi
- Homogenization vs Layering (Staircase)
  - Prandtl, Batchelor:



$\nabla q \rightarrow 0$

    - PV homogenized
    - Mechanism: Shear dispersion $\tau \sim \Omega\, Re^{1/3}$

Forward enstrophy cascade



  - Phillips, McIntyre (+ many color pics):
- Key: Scale selection?

Physics of <u>bistability</u>?

➔ - Cahn-Hilliard models !?
- Mergers

Discussion I: 40 years after Hasegawa-Mima Future Directions for Plasma (MFE Theory) – GFD Interactions

- Mesoscopic Pattern Competition
  - Eddy (streamer) vs Zonal flow – nonlinear evolution
  - Zonal flow <u>scale</u>:
    - Screening, neoclassical, nonlinear

      → Plasma
    - Mergers → GFD, Difference observation of mergers?
- "General Circulation" states and transitions
  - GFD: Jets + Hadley flow states, variability

    symmetry asymmetric
  - Tokamaks:
    - Symmetry breaking;
    - V_ϕ, V_θ states

ITB $V'_{E\times B}$ $V_\phi(r)$ radial force balance

Discussion I: 40 years after Hasegawa-Mima Future Directions for Plasma (MFE Theory) – GFD Interactions

- Boundary Layers, Dynamics
  - LH Transition, Pedestal – MFE
  - Western Boundary Layer
    - Gulf Stream
    - Kuroshio
- Multi-Scale Problem
  - ETG + ITG/DW
  - DW + AE
  - GFD + Micro-sources (Clouds)
- Microscopic Boundary Structures
  - Blobs, filaments – MFE
  - Tornados – GFD

Yoden (plenary) → QBO ←→ LCO: LH-transition

Zheng Yan (APS invited 2018): counter propagating waves in LH-transition

Discussion I: 40 years after Hasegawa-Mima
Future Directions for Plasma (MFE Theory) – GFD Interactions

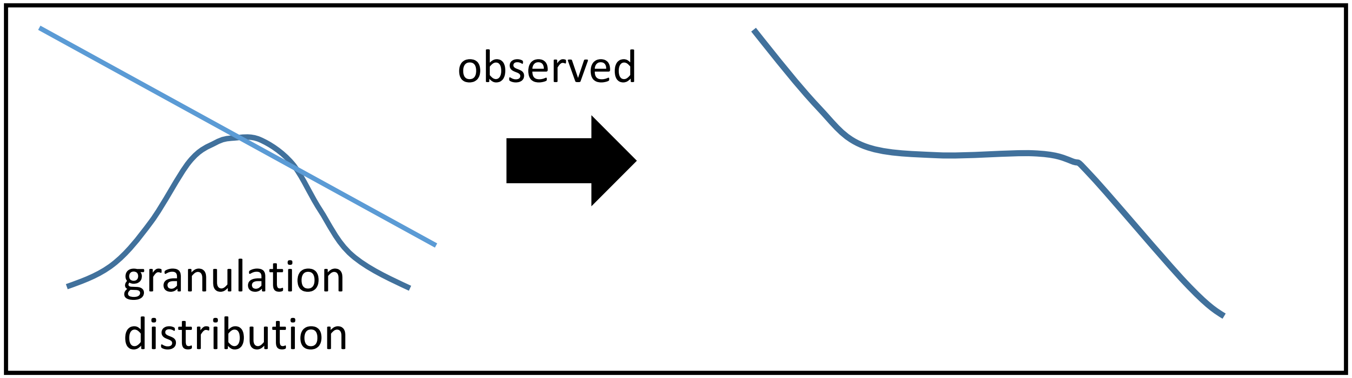
- Use of machine learning to deduce effective reduced models (Lathrop)

  c.f. Recent work by Ott group for Kuramoto-Sivashinsky system

  ➔ Can these lead to <u>understanding</u>?

Discussion II: Magnetic Self-Organization and Dynamo: Current Status, and Role of Boundary Effects

- Leaders: P. Chen, Cappello, M. Zhang
- Need improve remote measurement of magnetic helicity
- Footprint of convection zone structure on magnetics



- Magnetic helicity flux
  - transport physics ?!
  - $J_{\|}$ avalanches – RFP
- Inverse helicity cascade – fundamental !?

Discussion III: New Directions in Our Approach to Nonlinear Plasma Dynamics

- Leaders: Z.B. Guo, Kosuga

Nonlinear Phase Dynamics

1. What's nonlinear phase dynamics, why it is important?

In linear&quasilinear phase dynamics

$$\Theta(x,t) = -i\omega t + ikx \text{ with } |\partial_t ln\omega| \ll |\omega| \quad and \quad |\partial_x lnk| \ll |k|,$$

So there exists time&space scale separation and hence WKB, wave-action kinetics ….are valid.

BUT, in nonlinear phase dynamics

$$|\partial_t ln\omega| \geq |\omega| \quad and/or \quad |\partial_x lnk| \geq |k|.$$

So:

(a) the dynamical property of the concerned system may change, **qualitatively ;**

(b) new paradigm of coherent structure formation.

2. Rethinking wave-particle interaction from the viewpoint of phase couplings among particles

Example:

$$-\nabla^2\phi = 4\pi q n_0 \int \tilde{f} dv \xrightarrow[\text{transformation}]{\text{Fourier}} \frac{k^2}{4\pi q n_0} |\phi_k| e^{i\theta_\phi} = \int |f_k| e^{i\theta_f} dv$$

$|\phi_k|$ is an order parameter, describing the degree of synchronization among the phases($\theta_f(v)$) of different particles

Perspective on Relaxation: A Tale of Two Taylors

- Many commonalities between magnetic and flow relaxation apparent
- Common weak point is limitation of mean field theory
  → difficult to grapple with strong NL , non-Gaussian fluctuations

	Magnetic (JB)	Flow (GI)
concept	topology	symmetry
process	turbulent reconnection	PV mixing
players	tearing modes, Alfven waves	drift wave turbulence
mean field	EMF = $\langle \tilde{v} \times \tilde{B} \rangle$	PV Flux = $\langle \tilde{v}_r \tilde{q} \rangle$
constraint	$\int d^3x \mathbf{A} \cdot \mathbf{B}$ conservation	Potential Enstrophy balance
NL	Helicity Density Flux	Pseudomomentum Flux
outcome	B-profiles	zonal flow

- MFE theory has benefited greatly from long term interaction with GFD, AFD communities
- Focus on paradigmatic problems has been productive: Brute force parameter pushing, computation <u>not</u> so.
- Should continue focused C-D interaction in age of ITER, CFETR, SciDAC, etc.!